Question # 1 Without using the tables, find the value of :

(i) $\sin(-780^\circ)$ (ii) $\cot(-855^\circ)$ (iii) $\csc(2040^\circ)$

(iv) $\sec(-960)$ (v) $\tan(1110^\circ)$ (vi) $\sin(-300^\circ)$

Solution

(i) $\sin(-780^\circ)$

$= -\sin 780^\circ$

$= -\sin\left(8(90)+60\right)$

$= -\sin(60) = -\frac{\sqrt{3}}{2}$ $\because$ 780 is in the Ist quad.

(ii) $\cot(-855^\circ)$

$= -\cot 855^\circ$

$= -\cot\left(9(90)+45\right)$

$= -\left(-\tan 45^\circ\right)$ $\because$ 855 is in the IInd quad.

$= \tan 45^\circ = 1$

(iii) $\csc(2040^\circ)$

$= \csc\left(22(90)+60\right)$

$= -\csc(60)$ $\because$ 2040° is in the Ist quad.

$= -\frac{1}{\sin(60)}$

$= -\frac{1}{\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$

(iv) $\sec(-960)$

$= \sec(960)$

$= \sec\left(10(90)+60\right)$

$= -\sec 60^\circ$ $\because$ 960° is in the IIIrd quad.

$= -\frac{1}{\cos 60^\circ}$

$= -\frac{1}{1/2} = -2$

(v) $\tan(1110)$

$= \tan\left(12(90)+30\right)$

$= \tan(30) = \frac{1}{\sqrt{3}}$ $\because$ 1110° is in the Ist quad

(vi) $\sin(-300)$

$= -\sin(300)$

$= -\sin\left(3(90)+30\right)$

$= -(-\cos 30^\circ)$ $\because$ 300° is in the IIIrd quad.

$= \cos 30^\circ = \frac{\sqrt{3}}{2}$

Question # 2

Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.

(i) $\sin 196^\circ$ (ii) $\cos 147^\circ$ (iii) $\sin 319^\circ$ (iv) $\cos 254^\circ$

(v) $\tan 294^\circ$ (vi) $\cos 728^\circ$ (vii) $\sin(-625^\circ)$ (viii) $\cos(-435^\circ)$

Solution

(i) $\sin 196^\circ$

$$= \sin(180+16)$$
$$= \sin 180^\circ \cos 16^\circ + \cos 180^\circ \sin 16^\circ$$
$$= (0)\cos 16^\circ + (-1)\sin 16^\circ$$
$$= -\sin 16^\circ$$

(ii) $\cos 147^\circ$

$$= \cos(180-33)$$
$$= \cos 180^\circ \cos 33^\circ + \sin 180^\circ \sin 33^\circ$$
$$= (-1)\cos 33^\circ + (0)\sin 33^\circ$$
$$= -\cos 33^\circ$$

(iii) $\sin 319^\circ$

$$= \sin(360-41)$$
$$= \sin 360^\circ \cos 41^\circ - \cos 360^\circ \sin 41^\circ$$
$$=$$
$$=$$

(iv) $\cos 254^\circ$

$$= \cos(270-16)$$

(v) $\tan 294^\circ$

$$= \frac{\sin 294^\circ}{\cos 294^\circ}$$
$$= \frac{\sin(270+24)}{\cos(270+24)}$$
$$= \frac{\sin 270^\circ \cos 24^\circ + \cos 270^\circ \sin 24^\circ}{\cos 270^\circ \cos 24^\circ - \sin 270^\circ \sin 24^\circ}$$
$$= \frac{(-1)\cos 24^\circ + (0)\sin 24^\circ}{(0)\cos 24^\circ - (-1)\sin 24^\circ}$$
$$= \frac{-\cos 24^\circ + 0}{0 + \sin 24^\circ}$$
$$= \frac{-\cos 24^\circ}{\sin 24^\circ}$$
$$= -\cot 24^\circ$$

2nd Method:

$$\tan 294^\circ$$
$$= \tan(270+24)$$
$$= \frac{\tan 270^\circ + \tan 24^\circ}{1-\tan 270^\circ \tan 24^\circ}$$
$$= \frac{\tan 270^\circ\left(1+\frac{\tan 24^\circ}{\tan 270^\circ}\right)}{\tan 270^\circ\left(\frac{1}{\tan 270^\circ}-\tan 24^\circ\right)}$$
$$= \frac{\left(1+\frac{\tan 24^\circ}{\infty}\right)}{\left(\frac{1}{\infty}-\tan 24^\circ\right)}$$
$$= \frac{(1+0)}{(0-\tan 24^\circ)}$$
$$= -\frac{1}{\tan 24^\circ} = -\cot 24^\circ$$

(vi) $\cos 728^\circ$

$$= \cos(720+8)$$

$\downarrow$

(vii) $\sin(-625^\circ)$

$$= -\sin 625^\circ$$
$$= -\sin(630-5)$$
$$= -\left(\sin 630^\circ \cos 5^\circ - \cos 630^\circ \sin 5^\circ\right)$$
$$= -\left((-1)\cos 5^\circ - (0)\sin 5^\circ\right)$$
$$= -\left(-\cos 5^\circ - 0\right) = \cos 5^\circ$$

(viii) $\cos(-435^\circ)$

$$= \cos 435^\circ$$
$$= \cos(450-15)$$

$\downarrow$

Question # 3 Prove the following:

(i) $\sin(180+\alpha)\sin(90-\alpha) = -\sin\alpha\cos\alpha$

(ii) $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \frac{1}{2}$

(iii) $\sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ = 0$

(iv) $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$

Solution

(i) L.H.S

$= \sin(180+\alpha)\sin(90-\alpha)$

$= (\sin 180^\circ \cos\alpha + \cos 180^\circ \sin\alpha)(\sin 90^\circ \cos\alpha - \cos 90^\circ \sin\alpha)$

$= ((0)\cos\alpha + (-1)\sin\alpha))((1)\cos\alpha - (0)\sin\alpha)$

$= (0 - \sin\alpha))(\cos\alpha - 0)$

$= -\sin\alpha\cos\alpha$ = R.H.S

(ii) L.H.S

$= \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ$

$= \sin(720+60)\ \sin(450+30)\ +\ \cos(90+30)\ \sin 30^\circ$

$= \sin(2\times 360+60)\ \sin(5\times 90+30)\ +\ \cos(90+30)\ \sin 30^\circ$

$= \sin 60^\circ\ \cos 30\ +\ (-\sin 30)\ \sin 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ = R.H.S

(iii) L.H.S

$= \sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ$

$= \cos(270+36) + \cos(270-36) + \cos(180-18) + \cos 18^\circ$

$= \cos 270^\circ \cos 36^\circ - \sin 270^\circ \sin 36^\circ + \cos 270\cos 36 + \sin 270\cos 36$
$+ \cos 180^\circ \cos 18^\circ + \sin 180^\circ \sin 18^\circ + \cos 18^\circ$

$= (0)\cos 36^\circ - (-1)\sin 36^\circ + (0)\cos 36^\circ + (-1)\sin 36^\circ$
$+ (-1)\cos 18 + (0)\sin 18 + \cos 18^\circ$

$= \sin 36^\circ - \sin 36^\circ - \cos 18^\circ + \cos 18^\circ$

$= 0$ = R.H.S

(iv) L.H.S

$= \cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ$

$= \cos(360-30)\ \sin(6\times 90+60)\ +\ \cos(90+30)\ \sin(90+60)$

$= (\cos(-30^\circ))\ (-\sin 60)\ +\ (-\sin 30)\ (\cos 60^\circ)$

$= (\cos 30^\circ)(-\sin 60)\ +\ (-\sin 30)\ (\cos 60^\circ)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$\because$ 600° is in the IIIrd quad
$\because$ 120° is in the IInd quad
$\because$ 150° is in the IInd quad

$= -\frac{3}{4} - \frac{1}{4}$

$= -\frac{4}{4} = -1$ = R.H.S

Question # 4 Prove that

(i) $$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\,\text{cosec}(2\pi-\theta)}=\cos\theta$$

Solution

(i) L.H.S

$$=\frac{\sin^2(\pi+\theta)\quad\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\,\text{cosec}(2\pi-\theta)}$$

First we calculate

$$\begin{aligned}\sin(\pi+\theta) &= \sin\pi\cos\theta+\cos\pi\sin\theta\\ &=(0)\cos\theta+(-1)\sin\theta\\ &=0-\sin\theta=-\sin\theta\\ \Rightarrow \sin^2(\pi+\theta) &=(-\sin\theta)^2\end{aligned}$$

$$\tan\left(\frac{3\pi}{2}+\theta\right)=\tan\left(3\cdot\frac{\pi}{2}+\theta\right)=-\cot\theta \quad \because\ \frac{3\pi}{2}+\theta \text{ is in the IVth quad}$$

$$\cot\left(\frac{3\pi}{2}-\theta\right)=\cot\left(3\cdot\frac{\pi}{2}-\theta\right)=\tan\theta \quad \because\ \frac{3\pi}{2}-\theta \text{ is in the IIIrd quad}$$

$$\Rightarrow \cot^2\left(\frac{3\pi}{2}-\theta\right)=(\tan\theta)^2$$

$$\csc(2\pi-\theta)=\csc(-\theta)=-\csc\theta$$

$$\begin{aligned}\cos(\pi-\theta) &= \cos\pi\cos\theta+\sin\pi\sin\theta\\ &=(-1)\cos\theta+(0)\sin\theta\\ &=-\cos\theta+0=-\cos\theta\\ \Rightarrow \cos^2(\pi-\theta) &=(-\cos\theta)^2\end{aligned}$$

$$=\frac{(-\sin\theta)^2(-\cot\theta)}{(\tan\theta)^2(-\cos\theta)^2(-\csc\theta)}$$

$$=\frac{\sin^2\theta\,(-\cot\theta)}{\tan^2\theta\,\cos^2\theta\,(-\csc\theta)}$$

$$=\frac{\sin^2\theta\,\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta}{\cos^2\theta}\,\cos^2\theta\,\frac{1}{\sin\theta}}$$

$$=\frac{\sin\theta\cos\theta}{\sin\theta}$$

$$=\cos\theta = \text{R.H.S}$$

Question # 4 Prove that

(ii) $\dfrac{\cos(90^\circ+\theta)\ \sec(-\theta)\ \tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\ \sin(180^\circ+\theta)\ \cot(90^\circ-\theta)}=-1$

Solution

(ii) L.H.S

$$=\frac{\cos(90^\circ+\theta)\ \sec(-\theta)\ \tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\ \sin(180^\circ+\theta)\ \cot(90^\circ-\theta)}$$

First we calculate

$\cos(90+\theta)=-\sin\theta$

$\because\ 90+\theta$ is in the IInd quad.

$\sec(-\theta)=\sec\theta$

$\tan(180-\theta)=\tan\left(2(90)-\theta\right)=-\tan\theta$

$\because\ 180-\theta$ is in the IInd quad.

$\sec(360-\theta)=\sec(-\theta)=\sec\theta$

$\sin(180+\theta)=\sin\left(2(90)+\theta\right)=-\sin\theta$

$\because\ 180+\theta$ is in the IIIrd quad.

$\cot(90-\theta)=\tan\theta$

$\because\ 90-\theta$ is in the 1st quad.

$$=\frac{(-\sin\theta)\sec\theta\ (-\tan\theta)}{\sec\theta\ (-\sin\theta)\ (-\tan\theta)}$$

$=1$

= R.H.S

Question # 5 If α, β, γ are the angles of a triangle ABC, then prove that

(i) $\sin(\alpha+\beta)=\sin\gamma$ (ii) $\cos\left(\frac{\alpha+\beta}{2}\right)=\sin\frac{\gamma}{2}$

(iii) $\cos(\alpha+\beta)=\cos\gamma$ (iv) $\tan(\alpha+\beta)+\tan\gamma=0$

Solution

Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma=180$$
$$\Rightarrow \alpha+\beta=180-\gamma$$

(i)
$$\begin{aligned}\text{L.H.S} &= \sin(\alpha+\beta)\\ &= \sin(180-\gamma)\\ &= \sin 180\cos\gamma-\cos 180\sin\gamma\\ &= (0)\cos\gamma-(-1)\sin\gamma\\ &= 0+\sin\gamma\\ &= \sin\gamma = \text{R.H.S}\end{aligned}$$

(ii)
$$\begin{aligned}\text{L.H.S} &= \cos\left(\frac{\alpha+\beta}{2}\right)\\ &= \cos\left(\frac{180-\gamma}{2}\right)\\ &= \cos\left(\frac{180}{2}-\frac{\gamma}{2}\right)\\ &= \cos\left(90-\frac{\gamma}{2}\right)\\ &= \cos 90\cos\frac{\gamma}{2}+\sin 90\sin\frac{\gamma}{2}\\ &= (0)\cos\frac{\gamma}{2}+(1)\sin\frac{\gamma}{2}\\ &= 0+\sin\frac{\gamma}{2}\\ &= \sin\frac{\gamma}{2} = \text{R.H.S}\end{aligned}$$

Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma=180$$
$$\Rightarrow \alpha+\beta=180-\gamma \qquad \Rightarrow \frac{\alpha+\beta}{2}=\frac{180-\gamma}{2}$$

Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma=180 \qquad \Rightarrow \alpha+\beta=180-\gamma$$

(iii)
$$\begin{aligned}\text{L.H.S} &= \cos(\alpha+\beta)\\ &= \cos(180-\gamma)\\ &= \cos 180\cos\gamma+\sin 180\sin\gamma\\ &= (-1)\cos\gamma+(0)\sin\gamma\\ &= -\cos\gamma+0\\ &= -\cos\gamma = \text{R.H.S}\end{aligned}$$

(iv)
$$\begin{aligned}\text{L.H.S} &= \tan(\alpha+\beta)+\tan\gamma\\ &= \tan(180-\gamma)+\tan\gamma\\ &= \frac{\tan 180-\tan\gamma}{1+\tan 180\tan\gamma}+\tan\gamma\\ &= \frac{(0)-\tan\gamma}{1+(0)\tan\gamma}+\tan\gamma\\ &= \frac{-\tan\gamma}{1+0}+\tan\gamma\\ &= -\tan\gamma+\tan\gamma\\ &= 0 = \text{R.H.S}\end{aligned}$$

Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma=180 \qquad \Rightarrow \alpha+\beta=180-\gamma$$